

Q.(2). $\frac{d^2y}{dx^2} + \frac{2}{x} \cdot \frac{dy}{dx} + n^2 y = 0$

Like Q(1) we have $u = \frac{1}{x}$

and symbolic form of the given equation

$$(D^2 + n^2) u = 0$$

a.e. is $D^2 + n^2 = 0 \Rightarrow D^2 = -n^2 = i^2 n^2$

$$\therefore D = \pm in$$

$$\begin{aligned}\therefore C.F. &= e^{0 \cdot x} [C_1 \cos(nx) + C_2 \overset{\sin(nx)}{\text{sin}}(nx)] \\ &= C_1 \cos(nx) + C_2 \sin(nx)\end{aligned}$$

\therefore The complete solution of the give equation is

$$y = uv = \frac{1}{x} [C_1 \cos(nx) + C_2 \sin(nx)]$$

Q.(3). $\frac{d^2y}{dx^2} - \frac{2}{x} \cdot \frac{dy}{dx} + \left(n^2 + \frac{2}{x^2}\right)y = 0$

Here, $P = -\frac{2}{x}$, $Q = n^2 + \frac{2}{x^2}$ and $R = 0$

In order to remove the first derivative, we choose u such that

$$u = e^{-\frac{1}{2} \int P dx} = e^{-\frac{1}{2} \int (-\frac{2}{x}) dx} = e^{\int \frac{1}{x} dx}$$

$$= e^{\log x} = x$$

$$\therefore \boxed{u = x}$$

Taking $y = uv$ as complete solution of the given equation. The given equation gives

$$\frac{d^2y}{dx^2} + Iy = S \dots \dots \dots \dots \quad (1)$$

where $I = Q - \frac{1}{2} \frac{dP}{dx} - \frac{1}{4} P^2$

$$= n^2 + \frac{2}{x^2} - \frac{1}{2} \left(\frac{2}{x^2} \right) - \frac{1}{4} \cdot \frac{4}{x^2}$$

$$= n^2 + \frac{2}{x^2} - \frac{1}{x^2} - \frac{1}{x^2} = n^2$$

$$\therefore [I = n^2] \text{ and } [S = 0]$$

Hence, the complete solution of the given eqn. be

$$y = uy = x [c_1 \cos(nx) + c_2 \sin(nx)]$$

[According to Q. (2)]

Q.(4). Solve. $\frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 3)y = e^{x^2}$

Solution:— Comparing the given equation with the standard equation

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = R$$

We have

$$P = -4x, Q = (4x^2 - 3), R = e^{x^2}$$

In order to remove the first derivative of the given equation, we choose u such that

$$u = e^{-\frac{1}{2} \int P dx} = e^{-\frac{1}{2} \int (-4x) dx} = e^{2 \int x dx} = e^{x^2}$$

$$\therefore [u = e^{x^2}]$$

Taking $y = uv$ as complete solution of the given equation. The given equation can be transformed to normal form

$$\frac{d^2u}{dx^2} + Iu = S \dots \dots \dots \dots \dots \quad (1)$$

$$\begin{aligned} \text{where } I &= Q - \frac{1}{2} \frac{dp}{dx} - \frac{1}{4} p^2 \\ &= 4x^2 - 3 - \frac{1}{2}(-4) - \frac{1}{4} \cdot 16x^2 \\ &= 4x^2 - 3 + 2 - 4x^2 = -1 \\ \therefore \boxed{I = -1} \end{aligned}$$

$$\text{and } S = R/u = \frac{e^{x^2}}{e^{x^2}} = 1. \quad \therefore \boxed{S = 1}$$

Now, equation (1) becomes

$$\frac{d^2u}{dx^2} - u = 1$$

Symbolic form of the above equation is

$$(D^2 - 1)u = 1$$

A. e. is given by

$$(D^2 - 1) = 0 \Rightarrow D^2 = 1 \quad \therefore D = \pm 1$$

$$\text{Then, C.F.} = C_1 e^x + C_2 e^{-x}$$

$$\text{Now, P.I.} = \frac{1}{D^2 - 1} \cdot 1 = (D^2 - 1)^{-1} \cdot 1 = -1$$

$$\left[1 \cdot \frac{1}{D^2 - 1} \cdot e^{0x} = 1 \cdot \frac{1}{0^2 - 1} \cdot e^{0x} = -1 \right]$$

$$\therefore v = C.F. + P.I.$$

$$= C_1 e^x + C_2 e^{-x} - 1$$

Hence, the complete solution of the given equation be

$$y = u v = e^{x^2} [C_1 e^x + C_2 e^{-x} - 1] \rightarrow \text{Answer.}$$

$$\text{Q.}(5). \text{ Solve: } \frac{d^2y}{dx^2} - (2\tan x) \frac{dy}{dx} + 5y = \sec x \cdot e^x.$$

Solution: — Comparing the given equation to the standard equation

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = R$$

we get

$$P = -2\tan x, \quad Q = 5, \quad R = \sec x \cdot e^x$$

In order to remove the first derivative of the given equation, we choose u such that

$$u = e^{\int P dx} = e^{\int (-2\tan x) dx} = e^{\log(\sec x)} = \sec x$$

$$\therefore \boxed{u = \sec x}$$

Taking $y = u v$ as complete solution of the given equation. The given equation can be transformed to normal form:

$$\frac{d^2v}{dx^2} + I v = \dots \dots \dots \quad (1)$$

where $I = Q - \frac{1}{2} \frac{dP}{dx} - \frac{1}{4} P^2$ Page - 18

$$= 5 - \frac{1}{2}(-2) \cdot \sec^2 x - \frac{1}{4} \cdot 4 \tan^2 x$$

$$= 5 + \sec^2 x - \tan^2 x = 5 + \sec^2 x - (\sec^2 x - 1) = 6$$

$$\therefore \boxed{I = 6}$$

and $S = R/u = \frac{\sec x \cdot e^x}{\sec x} = e^x \quad \therefore \boxed{S = e^x}$

Now, equation (1) $\Rightarrow \frac{d^2 u}{dx^2} + 6u = e^x$

In symbolic form

$$D^2 u + 6u = e^x \Rightarrow (D^2 + 6)u = e^x$$

$$\therefore \text{Its a.e. is } D^2 + 6 = 0 \Rightarrow D^2 = -6 = i^2 6 \quad \therefore D = \pm i\sqrt{6}$$

i.e. $D = 0 \pm i\sqrt{6}$

Then, C.F. $= e^{0 \cdot x} [C_1 \cos \sqrt{6} \cdot x + C_2 \sin \sqrt{6} \cdot x]$

$$= C_1 \cos \sqrt{6} x + C_2 \sin \sqrt{6} x$$

and P.I. $= \frac{1}{D^2 + 6} \cdot e^{0 \cdot x} = \frac{1}{i^2 + 6} \cdot e^x = \frac{1}{7} e^x$

$$\therefore u = C.F. + P.I. = C_1 \cos \sqrt{6} x + C_2 \sin \sqrt{6} x + \frac{1}{7} e^x$$

Thus, the complete solution of the given equation

be $y = \sec x [C_1 \cos \sqrt{6} x + C_2 \sin \sqrt{6} x + \frac{1}{7} e^x] \rightarrow \text{Answer}$

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